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Module : Nuclear Models -3

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Description of Module	
Subject Name	Physics
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Contents of this Unit

1. Applications of Liquid Drop Model

The *Bethe-Weizsacker* Mass formula is given as

$$B(A,Z) = a_V A - a_S A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_A \frac{(N-Z)^2}{A} \pm \delta + \eta$$

On rearranging it and solving the equation for minimum mass, we get the mass parabola for a given isobar (A =constant) and has the lowest point at Z= Z_0, and it would give the value of Z for most stable isobar

$$Z_0 = -\frac{\beta}{2\gamma} \approx \frac{A/2}{1 + \frac{1}{4}(a_C / a_A) A^{2/3}}$$

Because of the presence of pairing energy term δ in the mass equation, the solution fall into two categories according to whether even-A or odd-A nuclei is taken into consideration. For odd-A nuclei the paring term (δ) is zero, leading to a single parabola for both e-o and o-e nuclei

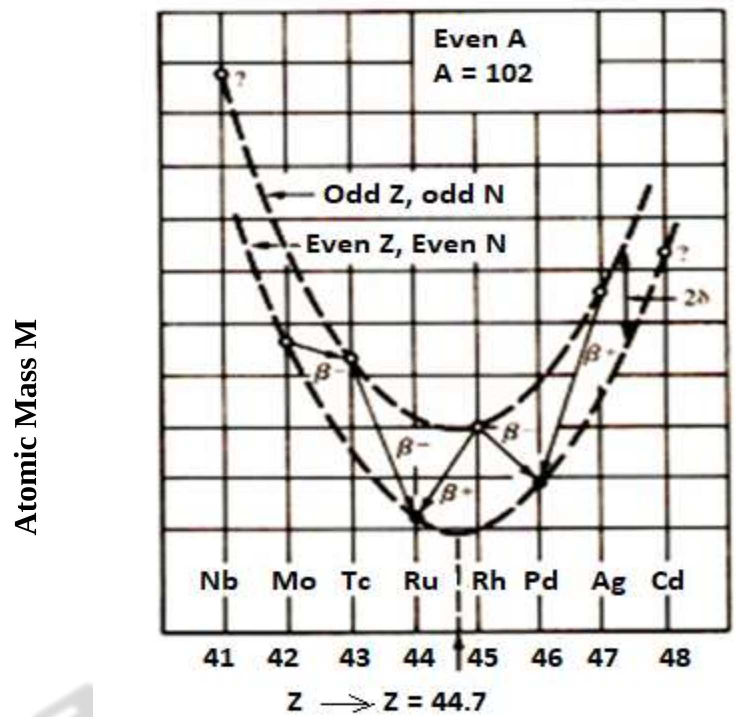

Fig. 1: Mass parabola for even-A nuclei. The value corresponding to $Z=Z_0$, indicates the most stable isobar

For **even-A nuclei** we get the following two conclusions

(1) No stable odd-odd nucleus is found. The change of mass with *Z* makes two parabolas narrower and with steeper sides. However, exceptions are there such as $^{2}H_1$, $^{6}Li_3$, $^{10}B_5$, and $^{14}N_7$.

(2) Many *e-e* nuclei can have more than one stable isobar. Examples are ^{40}K, ^{64}Cu.

2. Stability of Nucleus

Liquid drop model can also be used to find out the whether free nucleon decay is possible in case of β-decay or not. The β decay occurs by three modes, negative-beta decay (β^-), positive beta decay (β^+), and electron capture (EC).

2.1 Conditions for β decay

The cause of the instability that leads to β-decay is an excess of energy if there is a way of getting rid of this excess energy, then the decay will take place.

Therefore let us consider the case of energy balance in β decay

β^- Decay

The energy balance equation for negative beta decay is

$M(Z,A) = M(Z+1, A)\, c^2 - m_e c^{\,2} + Q$

Where *M is* the *nuclear mass* and *Q* is the net energy released

The reaction is possible only when it must satisfied the condition

$$Q > 0$$

Therefore the above equation becomes

$$Q = [M(Z, A) - M(Z+1, A) - m_e]c^2 > 0$$

It means for β^- decay to take place it is sufficient for the parent atom to have a mass greater than that of the daughter atom.

For β^+ decay the condition becomes

$$Q = [M(Z, A) - M(Z-1, A) - m_e]\, c^2 > 0$$

For Electron capture (EC)

$$Q_{EC} = [M\,(Z, A) - M\,(Z - 1, A\,) + m_e]\,c^2 > 0$$

2.2 Free Nucleon Decay

Free Neutron Decay

A free neutron when undergoes β-decay, it has half-life of 898 seconds (τ=898 seconds).

The reaction equation of neutron decay in β-decay is

$$n \rightarrow p + e^- + \bar{\nu}_e$$

Q-value of this reaction is

$Q_\beta = [M_n - (M_p + m_e)]\, C^2$

$= [939.573 - (938.791 + 0.511)]$ MeV

$=$ **0.782 MeV > 0**

Hence, we found that Q-value of this reaction is 0.782 MeV which is positive, so the condition for this reaction to go is fulfilled, hence this reaction is energetically possible and free decay of neutron is possible.

Free Proton Decay

Similarly, we can check whether free decay of proton is possible or not. Reaction equation for free proton can be represented as

$$p \rightarrow n + e^{+} + \nu_e$$

Q-value of this reaction is

$Q_\beta = [M_p - (M_n + m_e)]\, C^2$

$= [938.791 - (939.573 + 0.511)]$ MeV

$= \mathbf{-1.293\ MeV < 0}$

Since, Q-value for this reaction turns out to be negative so this reaction is energetically not possible, it means that decay of free proton is not possible. It has good implications on the stability of protons which is must for existence of universe.

Electron Capture in a Hydrogen Atom

Reaction equation for electron captures is written as

$$e^{-} + p \rightarrow n + \nu_e$$

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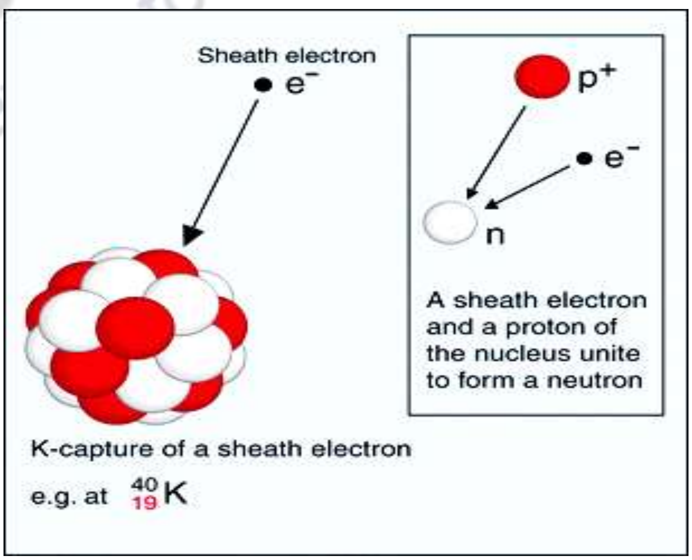

Fig. 2: Electron capture in an atom

Q-value of the above reaction is

$Q_\beta = [(M_p+m_e) - M_n]\, C^2$

$= [(938.791+0.511) - 939.573]$ MeV

$= - 0.271$ MeV <0

So, this reaction is also not energetically possible as Q-value is negative

Finally, we can summarize all the energy conditions in β-decay in terms of nuclear masses as

$\beta^-: \quad (Z,A) \Rightarrow (Z+1,A) + e^- + \bar{\nu}_e$ $\qquad$ $\mathbf{Q_\beta = [M(Z,A) - M(Z+1, A) - m_e]c^2 > 0}$

$\beta^+: \quad (Z,A) \Rightarrow (Z-1,A) + e^+ + \nu_e$ $\qquad$ $\mathbf{Q_\beta = [M(Z,A) - M(Z-1, A) - m_e]c^2 > 0}$

$\mathrm{EC}: \quad (Z,A) + e^- \Rightarrow (Z-1,A) + \nu_e$ $\qquad$ $\mathbf{Q_{EC} = [M(Z,A) - M(Z-1, A) + m_e]c^2 > 0}$

If the decay happens, the excess energy is shared as kinetic energy among the products to conserve linear momentum.

3. Fission

The semi-empirical mass formula can also be used to explain the phenomenon of nuclear fission and will be discussed here.

Nuclear fission is the result of the competition between the Coulomb energy and the surface tension. It is a special type of nuclear reaction in which an excited compound nucleus breaks up generally into two fragments of comparable mass numbers and atomic numbers. Fission usually occurs amongst the isotopes of the heaviest elements, e.g., uranium, thorium etc.

Nuclear fission was first discovered by the two German chemists Otto Hahn and F. Strassmann in 1939.

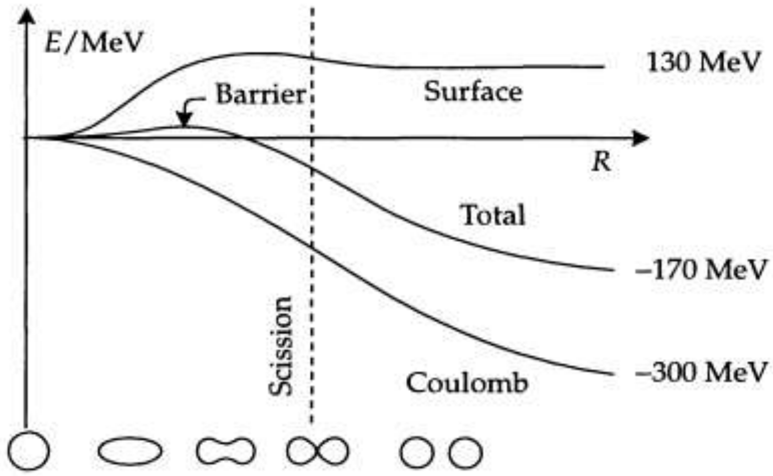

Fig. 3: Representation of fission in a heavy nucleus using liquid drop model.

3.1 Energy Released in Fission

We can use the expression for the binding energy given by the liquid drop model to calculate energy released in fission.

Let nucleus (Z, A) in fission splits into two nuclei (Z_1, A_1) & (Z_2, A_2)

Clearly $Z = Z_1 + Z_2$ and $A = A_1 + A_2$

From liquid drop model writing the atomic masses in terms of the semi-empirical mass formula and ignoring the asymmetry energy and pairing energy terms. The energy released, E_R, is the difference between the final and the initial binding energies.

$$E_R = C_1\,(A_1 + A_2 - A) - C_2\,(A_1^{2/3} + A_2^{2/3} - A^{2/3}) - C_3\,\left(\frac{Z_1^2}{A_1^{1/3}} + \frac{Z_2^2}{A_2^{1/3}} - \frac{Z^2}{A^{1/3}}\right)$$

Volume energies-cancel | **Surface energies-different** | **Coulomb energies - different**

In the above equation first term is the volume energy which is cancels here, second term is the surface energy term which is different, and third term is the Coulomb energy term which is also different. So only surface and Coulomb energy terms appears in the energy released equation and there is a competition between only these terms, which is responsible for the splitting of heavy nuclei when the Coulomb energy exceeds the surface energy.

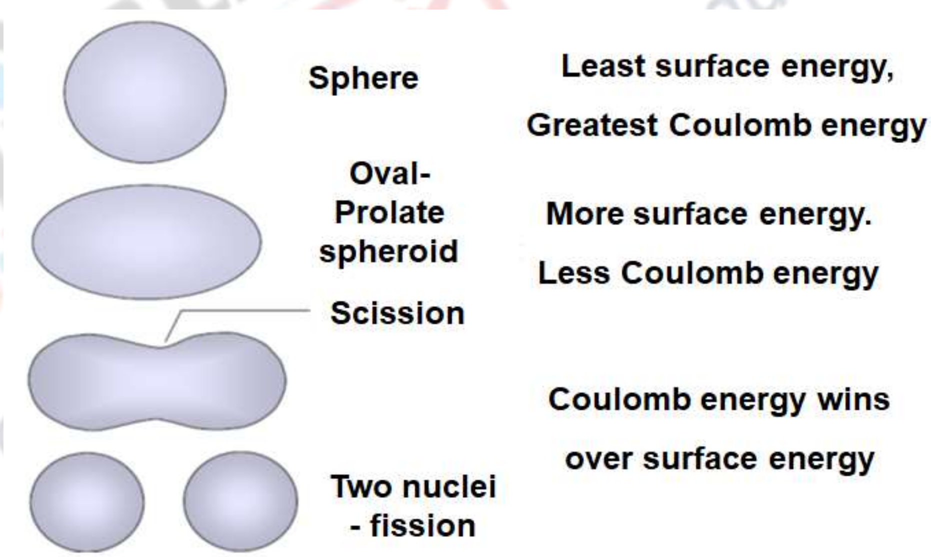

Fig. 4: Various stages of deformation leading to the final splitting of a liquid drop.

3.2 Nuclear fission based on the liquid drop model

N. Bohr and J.A Wheeler put forward the theory of nuclear fission based on the liquid drop model of the nucleus.

As in case of liquid drop if mechanical vibrations are set up within liquid drop, it can lead to the breakup of the drop. In order to do this, energy must be supplied from outside by some source. Since it is assumed that an atomic nucleus behaves like a charged liquid drop, similar vibrations may also be achieved in it if it gains some excitation energy which is possible if, for some instance the nucleus absorbs a neutron. The vibrations set up in the nucleus will deform it due to which its surface energy E_S and electrostatic Coulomb energy E_C are both changed.

As the fission process take place, the splitting of the nucleus is preceded by severe deformation of the original nucleus. The surface force tend to restore the original shape, while the Coulomb forces have the effect of increasing the deformation, because the surface energy is a minimum for the sphere while the Coulomb energy decreases with increased deformation. After the various stages of deformation when the Coulomb energy exceeds over surface energy, it will lead to the final splitting of a liquid drop into two fragments as shown in fig. 4.

From the above equation we can see that fission becomes energetically possible as energy released E_R changes its value from negative to positive with increasing A (i.e. when E_R =0).

If we put A_1=A_2= A/2, then the condition of fission happens when: $\boldsymbol{\frac{Z^2}{A} \approx 0.7\ \frac{C_2}{C_3}}$

However, the actual value turns out to be differ from this value and the fission does not appear at this value due to Coulomb Barrier, although it becomes energetically possible because of the" Coulomb Barrier".

On substituting value for C_1 and C_2 and setting Z~ A/2, shows that as per liquid drop model (LDM), the nuclei heavier than **A~72** are unstable against fission. Thus for nuclei for which A > 72, spontaneous fission should be energetically possible. In reality, however, this does not actually happen. Nuclei only start to fission spontaneously when A reaches ~240. The reason for this is due to barrier penetration problem; there is little probability of the fission to take place.

Nucleus will only fission *spontaneously* if separation energy is near the top of CB (Coulomb Barrier). Which happens only when

$$\frac{Z^2}{A} \approx 2.1\,\frac{C_2}{C_3}$$

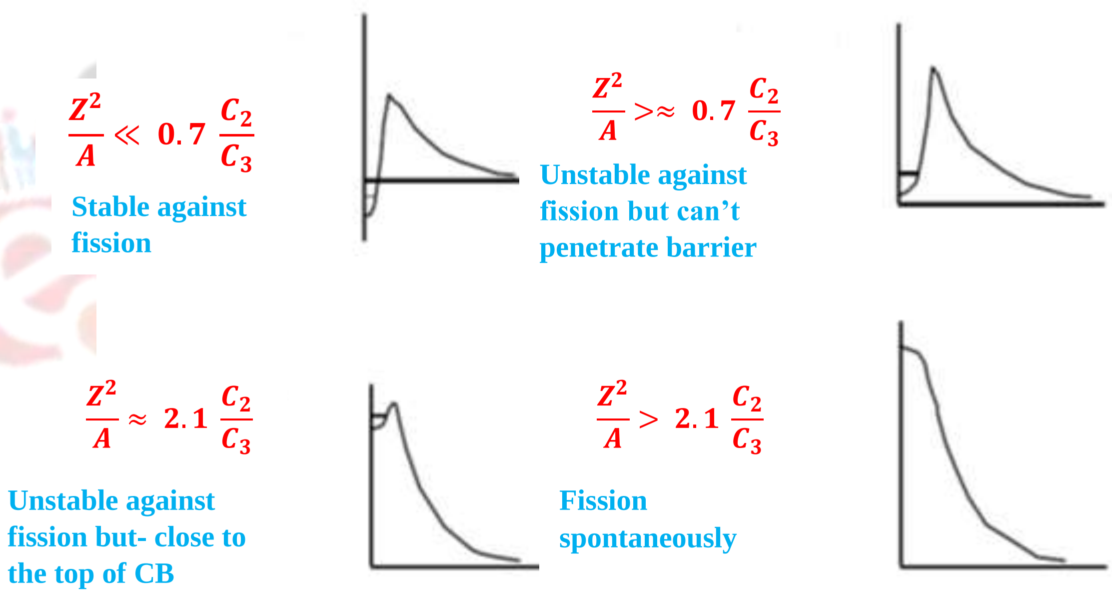

Fig. 5: Fission and the Coulomb barrier

Summary

The liquid drop model is able to explain many observed properties of the nucleus successfully. Using this model the average binding energy per nucleon curve can be fit well with accuracy (good to < 1%). The Coulomb term calculations obtained with the semi-empirical mass formula agree well with experimental observed values, and is able to explain the valley of stability well.

The decay of various unstable nuclei via selected modes can also be explained with the help of liquid drop model, and can explain the energetics of radioactive decays well. This model could also explain the process of fission and fusion well.